**MATRIX AND ITS APPLICATIONS**

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**CHAPTER ONE**

**INTRODUCTION AND LITERATURE REVIEW**

**1.1 BACKGROUND OF THE STUDY**

The introduction and development of the notion of a matrix and the subject of linear algebra followed the development of determinants, which arose from the study of coefficients of systems of linear equations. Leibnitz, one of the founder of calculus, used determinant in 1963 and Cramer presented his determinant based formula for solving systems of linear equation (today known as Cramer’s rule) in 1750.

The first implicit use of matrices occurred in Lagrange’s work on bilinear form in late 1700. Lagrange desired to characterize the maxima and minima of multi-variant functions. His method is now known as the method of Lagrange multipliers. In order to do this he first required the first order partial derivation to be 0 and additionally required that a condition on the matrix of second order partial derivatives holds; this condition is today called positive or negative definiteness, although Lagrange did not use matrices explicitly.

Gauss developed elimination around 1800 and used it to solve least square problem in celestial computations and later in computations to measure the earth and it’s surface (the branch of applied mathematics concerned with measuring or determining the shape of the earth or with locating exactly points on the earth’s surface is called Geodesy). Even though Gauss name is associated with this technique eliminating variable from system of linear equations there were earlier work on this subject.

Chinese manuscripts from several centuries earlier have been found that explains how to solve a system of three equations in three unknown by “Guassian” elimination. For years Gaussian elimination was considered part of the development of geodgesy, not mathematics. The first appearance of Gaussian-Jordan elimination in print was in a handbook on geodesy written by Wihelm Jordan. Many people incorrectly assume that the famous mathematician, Camille Jordan is the Jordan in “Gauss-Jordan elimination”.

For matrix algebra to fruitfully develop one needed both proper notation and proper definition of matrix multiplication. Both needs were met at about the same time in the same place. In 1848 in England, J.J Sylvester first introduced the term “matrix”, which was the Latin word for “womb” as a name for an array of numbers.

Matrix algebra was nurtured by the work of Arthur Cayley in 1855. Cayley studied multiplication so that the matrix of coefficient for the composite transformation ST is the product of the matrix S times the matrix T. He went on to study the algebra of these composition including matrix inverses. The famous Cayley-Hamilton theorem which asserts that a square matrix is a root of it’s characteristics’ polynomial was given by Cayley in his 1858 memoir on the theory of matrices. The use of single letter “A to represent a matrix was crucial to the development of matrix algebra. Early in the development, the formular det(AB) = det (A) det(B) provided a connection between matrix algebra and determinants. Cayley wrote “There would be many things to say about this theory of matrices which should, it seems to me, precede the theory of determinants”.

Mathematicians also attempted to develop for algebra of vectors but there was no natural definition of the product of two vectors that held in arbitrary dimensions. The first vector algebra that involved a non- commutative vector product (that is V x W need not equal W x V) was proposed by Hermann Grassman in his book – Ausedehnungslehre (1844). Grossmann’s text also introduced the product of a column matrix and a row matrix, which resulted in what is now called a simple or a rank one matrix. In the late 19th century the American mathematical physicist, Willard Gibbs published his famous treatise on vector analysis. In that treatise Gibbs represented general matrices, which he called dyadics as sum of simple matrices, which Gibbs called dyads. Later the physicist, P.A.M. Dirac introduced the term “bracket” for what we now call the “scalar product” of a “bar” (row) vector times a “ket” (column) vector and the term “ket-bra” for the product of a ket times a bra, resulting in what we now call a simple matrix, as above. Our convention of identifying column matrices and vector was introduced by physicists in the 20th century.

Matrices continued to be closely associated with linear transformations. By 1900, they were just a finite dimensional sub case of the emerging theory of linear transformations. The modern definition of a vector space was introduced by Peano in 1888. Abstract vector space whose elements were function soon followed. There was renewed interests in matrices, particularly on the numerical analysis of matrices, John Von Neumann and Herman Goldstein introduced condition numbers in analyzing round – off errors in 1947. Alan Turing and Von Neumann, the 20th century giants in the development of stored – program computers. Turning introduced the *LU* decomposition of a matrix in 1948. The L is a lower triangular matrix with I’s on the diagonal and the U is an echelon matrix. It is common to use LU decomposition in the solution of *n* sequence of systems of linear equations, each having the same co-efficient matrix. The benefit of the QR decomposition was realized a decade later. The Q is a matrix whose column are orthogonal vector and R is a square upper triangular invertible matrix with positive entities on its diagonal.

The QR factorization is used in computer algorithms for various computations, such as solving equations and find eigenvalues. Frobenius in 1878 wrote an important work on matrices on linear substitutions and bilinear forms, although he seemed unaware of Cayley’s work. However be proved important results in canonical matrices as representatives of equivalences classes of matrices. He cites Kronecker and Weiserstrases having considered special cases of his results in 1868 and 1874 respectively.

Frobenius also proved the general result that a matrix satisfies it’s characteristic equation. This 1878 paper by Frobenius also contains the definition of the rank of a matrix, which he used in his work on canonical forms and the definition of orthogonal matrices.

An axiomatic definition of a determinant was used by Weierstrass in his lectures and after his death, it was published in 1903 in the note on determinant theory. In the same year Kronecker’s lectures on determinants were also published after his death. With these two publications, the modern theory of determinants was in place but matrix theory took slightly longer to become a fully accepted theory. An important early text which brought matrices into their proper place within mathematics was introduction to higher algebra by Bocher in 1907. Turnbull and Aitken wrote influential text in the 1930s and Missky’s; “An introduction to linear algebra” in 1955 saw matrix theory to reach its present major role as one of the most important undergraduate mathematics topic.

### 1.2 SCOPE OF STUDY

In this study we are going to focus on m x n matrices of different order, i.e 2 x 3, 3 x2, 3 x3, etc. algebra of matrices, i.e. the different operation of addition, subtraction, scalar multiplication, matrix multiplication (under which we will consider power of matrices) and see if division is defined for matrices, determinant of different order starting with 2,3, etc.

Also of square matrix (under which we consider cofactors and adjoint) and the different properties of determinant; inverse of square matrix, product of a square matrix and it’s inverse and also special types of square matrix and the different applications of matrices.

#### 1.3 SIGNIFICANT OF STUDY

Matrices are key tools in linear algebra. One of the uses of matrices is to represent linear transformations, which are higher dimensional analogs of linear functions where matrix multiplication corresponds to composition of linear transformation which is used in computer graphics to project 3- dimensional space onto a 2- dimensional screen.

A major branch of numerical analysis is devoted to the development of efficient algorithms for matrix computations and for a square matrix, the determinant and inverse matrix (when it exists) govern the behaviour of solution of the corresponding system of the linear equations and eigenvalue and eigenvectors provide insight into the geometry of the associated linear transformation. The study of matrix is applicable to every aspect of human endeavour.

1.4 **TYPES OF MATRICES**

1.4.1 **Row Matrix**

A row matrix consists of 1 column only e.g. (3 2 4) is a row matrix of order 1 x 3.

1.4.2 **Column Matrix**

A column matrix is matrix having only one column.

e.g.  is a column matrix of order 3 x 1

So to conserve space in printing, a column matrix is sometimes written on one line with “curly” bracket e.g. (3 2 4) and is the same matrix as order 3 x 1.

1.4.3 **Single Element Matrix**

A single matrix number may be regarded as a matrix as a [x] matrix is having 1 row and 1 column.

**1.4.4 Double Suffix Matrix**

Each element in a matrix has its own particular address or location which can be defined by a system of double suffixes, the first indicating the row and the second the column, thus:

indicates element in the third row and element in the second column.

1.4.5 **Matrix Notation**

A whole matrix can be denoted by a single general element enclosed in brackets, or by a single letter printed in bold types. This is a very neat shorthand and saves much space, for example:

 can be denoted by or (a) or by A

1.5 **SPECIAL MATRICES**

1.5.1 **Square Matrix**

A square matrix is a matrix of order m x m meaning of the same number of rows and columns. e.g.

1 2 5

6 8 9 is a 3 x 3 matrix

1 7 4

A square matrix (aij) is symmetric of aij = aji

1 2 5 1 2 5

2 8 9 = 2 8 9

5 9 4 5 9 4

i.e. it is symmetrical about the leading diagonal.

Note: A = A

A square matrix (aij) is skew-symmetric of aij = -aji e.g.

1 2 5 -1 -2 -5

2 8 9 = - -2 -8 -9

5 9 4 -5 -9 -4

in this case A = -AT

**1.5.2 Diagonal Matrix**

A square matrix is called a diagonal matrix, if all its non-diagonal element are zero e.g.

1 0 0

0 3 0

0 0 4

1.5.3 **Unit or Identity Matrix**

A square matrix is called a unit matrix if all the diagonal elements are unity and non-diagonal elements are zero e.g.

(i) 1 0 0

0 1 0

0 0 1

(ii) 1 0

0 1

1.5.4 **Null Matrix or Law Matrix**

Any where in which the elements are zero is called a Zero Matrix or Null Matrix. e.g.

0 0 0

0 0 0

0 0 0

1.5.5 **Equal Matrix**

Two matrixes are said to be equal if:

(i) They are of the same order

(ii) The elements in the corresponding position are equal.

Thus of A = 2 3 B = 2 3

1 4 1 4

A=B

1.5.6 **Singular Matrix**

If the determinant of a matrix is zero, then the matrix is known as singular matrix e.g.

 then A is a singular matrix.

1.5.7 **Triangular Matrix (Echelon Form)**

A square matrix, all of whose element below the leading diagonals are zero is called an upper triangular matrix. A square matrix, all of whose elements above the leading diagonal are zero, is called a lower triangular matrix. e.g.

1 3 2 1 0 0

0 4 1 4 1 0

0 0 0 6 8 5

Upper triangular matrix Lower triangular matrix

1.5.8 **Orthogonal Matrix**

A square matrix A is called an orthogonal matrix, if the product of the matrix A and the transpose matrix A1 or ‘A’ is a unity matrix e.g.

A.AT = I

If 

1.5.9 **Non-Singular or Invertible Matrix**

A matrix A is called non-singular matrix if its inverse exist.





**How to get the inverse of matrix A:**

To get the inverse of matrix A the following rules must be observed or followed:

1. Interchange the two elements on the diagonal.
2. Take the negative of the other two elements.
3. Multiply the resulting matrix by  or equivalently, divide each element by . In case  = O, the matrix A is not vertible or non singular.

Expressing the process of inverting matrixes as a rule we do the following:

1. We get the minors of the matrix.
2. We sign the minors with the rule (-1) i+j to obtain the cofactors.
3. We transpose the cofactors
4. Multiply the result with the reciprocal of the determinant of the original matrix i.e. 

Using matrix 

where CT = B Adj A = CT

Consider A = 







1.5.10 **Conjugate of a Matrix**

Let 

then the conjugate of matrix A is Ă

Ă 

1.5.11 **Idempotent Matrix**

A matrix, such that A2 = A is called an idempotent matrix e.g.





1.5.12 **Periodic Matrix**

A matrix A will be called a periodic matrix, if AK+1 = A where K is a positive integer. If K is the least positive integer, for which AK+1 = A, then K is said to be periodic of A. if we choose K=1 we get A2 = A and we call it to be idempotent matrix.

1.5.13 **Nilpotent Matrix**

A matrix will be called a nilpotent matrix, if AK = 0 (null matrix) where K is a positive integer, if however   
is the least positive integer of which AK = 0, then K is the index of the nilpotent matrix.

A = ab b2 , A2 = ab b2 ab b2 = 0 0 = 0

-a2 -ab -a2 -ab -a2 -ab 0 0

1.5.14 **Involuntary Matrix**

A matrix A will be called an involuntary matrix, if A2 = I (unit matrix) since I2 = I always, it therefore means that a unit matrix is an involuntary matrix.

1.5.15 **Transpose of a Matrix**

In a given matrix A, we interchange the rows, and the corresponding columns, the new matrix obtained is called the transpose of matrix A and is denoted by A0 and A1 ­­e.g.



1.5.16 **MATRIX **

The transpose of the conjugate matrix A is denoted by 



1.5.17 **Unitary Matrix**

A square matrix A is said to be unitary of AT A = I





**Proof:** 

Where π is called the conjugate of matrix A.



Then A ∂ A = I







